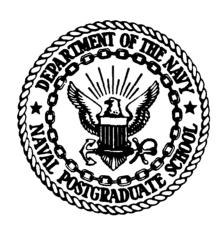


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NAVAL POSTGRADUATE SCHOOL Monterey, California



THESIS

ROUTING A USCG BUOYTENDER
TO SERVICE AIDS TO NAVIGATION:
A CASE OF THE TRAVELING SALESMAN PROBLEM

bv

Jon Michael Bechtle

September 1984

Thesis Advisor:

G. L. Lindsay

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The algorithm is programmed in Convergent Technologies FORTRAN for use on the Coast Guard Standard Terminal. Several problems are solved by the algorithm producing solutions that are optimal or nearly optimal. Arricved for public release; distribution unlimited.

Routing a USCG Buoytender to Service Aids to Navigation A Case of the Traveling Salesman Problem

Ьy

Jon M. Bechtle Lieutenant, United States Coast Guard B.S., U.S. Coast Guard Academy, 1978

Submitted in partial fulfillment of the requirements for the degree of

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from the

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I. INTRODUCTION

Cne of the U.S. Coast Guard's missions is maintaining aids to navigation in the waters of the United States. These aids require periodic servicing to ensure they are on station and they are showing their proper watch characteristics. The responsibility of maintaining many of these aids falls on the Coast Guard's fleet of buoytenders. A buoytender in its area of responsibility may have as many as 200 aids which it is required to maintain.

The buoytender has scheduled ATON runs (Aids TO Navigation) and emergency outages it must handle during a fiscal year. An emergency outage occurs when an aid is reported by a mariner to be showing improper characteristics or is missing from its station. These outages are handled shortly after they are reported. Several times during the year the buoytender plans an ATON run to handle the periodic servicing of aids, to ensure that they maintain their proper watch characteristics.

Prior to a scheduled run the tender will receive SANDS forms from the unit's district office for each of the aids which are due for service or relief. SANDS is a database where information is kept on a district's navigational aids. The district's aids to navigation branch identifies aids which are due for service or relief and notifies the tender. The district provides a list of aids which need to be serviced during the scheduled run and of work needed on each aid. The tender then makes plans accordingly to schedule and complete the work.

The number of aids serviced on a particular scheduled ATON run may be as few as 5 (a day of local work) to as many as 70 (an extended four-week trip servicing the west coast

of Alaska from the Aleutian Islands to Point Hope). The maintenance of the floating aids and fixed structures entails the travel of the tender from aid to aid. The tender's Crerations Officer usually plans the initial route to be travelled to service the aids due for routine maintenance and presents his proposal to the Commanding Officer, who may then modify this proposal.

There are several factors which go into the decision process to produce the desired route. Two of these factors are (1) which aids are scheduled for service, and (2) the distances between the aids. The desired route is usually the route of shortest distance which visits all the scheduled aids and returns the tender to her point of origin. This problem is a classic problem which operations analysts normally call 'the traveling salesman problem.' Present route selection methods are (1) traditional or previously followed routes and (2) routes created by Operations Department personnel sitting down with chart and rule and selecting a route.

It is the intent of this thesis to propose and implement a computer assisted approach as an alternative method for the huoytender route selection problem. The succeeding chapters of this paper contain a discussion of the traveling salesman problem and how it relates to the buoytender problem, a brief survey of possible solution methods to the problem, a description of the solution process of the selected method, and the results of the route selection program with conclusions.

II. NATURE OF THE PROBLEM

In this chapter the principles of the traveling salesman problem will be presented and it will be shown how the buoy-tender problem can be described as a traveling salesman problem. The discussion will include a general description, definitions about solution methods, and the solution requirements of the problem. Further, a description of the computing hardware on which the selected solution method is programmed is provided. This provides an idea of the capabilities of the hardware since the hardware has a bearing on the selection of the solution method.

A. DESCRIPTION OF THE TRAVELING SALESMAN PROBLEM

A brief description of the traveling salesman problem (or TSP) is as follows. A salesman has n cities he must visit. The salesman starts in one of the cities and must travel so that he enters and departs each of the other cities only once. Upon visiting the last city the salesman will return to the city from which he originally started his travels. The desired solution to the problem is the route the salesman can traverse which is of the shortest total distance possible.

It is usually assumed that a TSP can be associated with some n by n distance matrix. The elements of the matrix, d_{ij} , are distances from city i to city j, (i=1,...,n), (j=1,...,n), and where $d_{ij} \equiv \infty$, i=1,...,n. The traveling salesman is not allowed to leave city i and to return to city i in his tour.

A TSF can be associated with a graph, also. A city which the sales man must visit is represented by a node in

this graph. There is an arc (i,j) in the graph with length di for each node.

A distance matrix may be classified as sparse or dense. A sparse matrix will have a majority of its entries equal to infinity. The corresponding graph is sparse since it has few edges. A dense matrix is a matrix where a majority if not all its entries, with the exception of the die's, are less than infinity. The number of possible permutations of cities for a dense matrix is of order (n-1)!. Computation time and memory required for solution by total enumeration grows rapidly with the size of n due to the large number of possible solutions, each permutation of cities being a feasible tour and a possible solution.

The distance matrix used by the buoytender for route selection will be a dense matrix. With the large number of permutations or possible solutions to the problem, some means other than total enumeration must be used to solve the tuoytender problem.

The traveling salesman problem may be broken into two classes, symmetric and asymmetric. In a symmetric TSP the $d_{ij} = d_{ii}$ for all $i \neq j$, while in an asymmetric TSP this need not be true. Further, a TSP may or may not be required to satisfy the triangle inequality. Consider three cities i, j, k. The triangle inequality states that

 $dij \leq dik + d_{kj} \qquad \text{for all } i \neq j \neq k \neq i \,.$ The conditions which make a TSP asymmetric, the unequal distances between cities, may cause the problem to violate the triangle inequality.

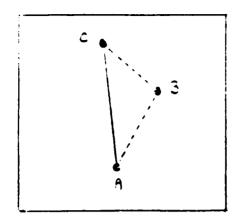
The TSP, both symmetric and asymmetric, belongs to the NP Complete class of problems [Ref. 1,2]. This has one major implication with respect to the complexity of solving the TSP: no polynomial-time algorithms are known or seem likely to be devised for exact solution of the TSP. Only exponential-time exact algorithms are known.

There have been many algorithms proposed for the solution of the TSP, each algorithm showing some advantages and disadvantages to its method of attacking and solving the Several of the published algorithms are tailored to a particular size of TSP. The TSP may be categorized by size as either small or large. A reference could not be found in the literature which would give a hard numerical value by which the size of a TSP could be judged, small TSF is generally considered to be less then 15 cities. A large ISP is often considered to be in the neighborhood of 45 or more cities. There are solution methods which solve the TSP exactly for small up to large problems. some methods which can solve some large problems exactly, but as a rule large problems are solved with heuristics or approximate methods.

The circumstances of the buoytender problem satisfy the conditions of a class of the traveling salesman problem. The buoytender problem can be characterized as a symmetric TSP, i.e., the distance traveled between any two aids is the same regardless of direction travelled by the tender. The size of buoytender problem will vary with the number of aids scheduled to be serviced on a specific ATON run; therefore the solution method for the buoytender problem will have to handle both small and large problems.

The triangle inequality as applied to the buoytender problem holds true, but how it holds needs some explanation. With all distances positive, the triangle inequality states that given three points A, B, and C, the distance from A to C must be less than or equal to the distance from A to B plus the distance from B to C (see Figure 2.1). In the buoytender's problem the straight-line distance from A to C may be less than (A to B) + (B to C), but some situations will require that A to C equal (A to B) + (B to C) (see Figure 2.1),

 $d_{AC} = d_{AB} + d_{BC}$.



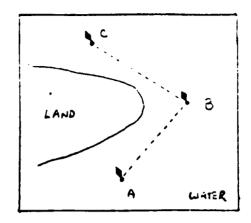


Figure 2.1 Triangle Inequality / Obstructed & to C Path

The buoytender may be unable to take the direct route from A to C due to shoal water or an obstruction. To get from A to C the tender will have to take the routes A to B and B to C. This is not an unusual situation for buoytenders since the aids being serviced cften mark shoal water or obstructions.

B. DESCRIPTION OF THE SOLUTION REQUIREMENTS

There are many proposed solution methods to the TSP and they may be divided into two groups: exact methods approximate or heuristic methods. An exact method or algorithm has the property that upon termination of the algorithm, the user will have the best possible or global optimum solution. Heuristics or approximate methods, on the other hand, terminate with a local optimum solution or the best solution found thus far. This local optimum solution may be the global optimum but usually there is no proof that the global optimum has been found. Heuristics or approximate methods solve by checking a subset of the numerous possible solutions and terminate when the subset searched for the best answer it contains, a specified time

limit for the run has been reached, or the memory capacity of the computing machine has been reached.

A further consideration in selecting a method to solve the TSP is that there should be very little interactive work for the user. The user interactive work necessary to operate the program should be understandable to a majority of the intended users, who in this case are the buoytender's Operations Officer or Navigator, and Ouartermaster By requiring the interactive work to be minimal personnel. and simple in nature the possiblity of operator introduced Further, if the interactive work is error will te reduced. of a simple nature. the program will be useable by a majority of the intended users without extensive training.

Although it is desired to obtain the global optimum, it is not mandatory. The computer route selection method is to be a tool used in planning, and the exact solution is not critical to the operation of the buoytender. By using heuristics, computer time and memory may be reduced since a heuristic generally searches only a subset of all possible If a computer solution can be produced which is tetter than the traditional route or a route produced by hand calculation, then a goal has been met, that goal being reducing the total distance traveled by the tender thereby saving fuel and time. The computer solution is then a viable alternative to be added to the present methods of route selection.

C. CCMPUTING HARDWARE

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The computer on which the selected solution method is programmed is the Coast Guard's C3 Standard Terminal. The reason for selecting this computer is that it is presently being installed throughout the Coast Guard as a primary unit-level computer. Most intend users should have access to a C3 Terminal.

The C3 Standard Terminal is a standalone 16 bit microcomputer. The terminal comes in two basic configurations, the Integrated Work Station (IWS) and the Application Work (AWS) . The IWS is a master or standalone unit equipped with a hard disk and 8 inch floppy mass storage system, a 300/1200 baud modem, a printer, and one of three RAM memory configurations - 256k, 384k, or 512k. may be programmed in BASIC, FORTRAN, COBOL, Pascal. The AWS is more limited in its capabilities than The AWS is limited to 256k RAM and must be the IWS. networked to a IWS station to access the peripherals. a program in RAM the AWS will operate as a computer, but does not have the power or computational speed of the IWS.

The solution method selected for programming will have to perform within the limitations of the C3's memory. memory will be a critical factor in selection of the solution method. Since the computer is a purchased standalone and not a leased system, CPU time should not be a critical factor. It is still desired, though, to keep CPU time from becoming excessive since the computer is also needed for other work. The computer could be allowed to run overnight (off duty hours) for large problems, and therefore a solution method requiring more than 12 hours to provide an acceptable solution would be excessive. A solution method which provides an answer in under an hour could be run during duty hours with very little impact on other users, particularly if it was run on a slave terminal.

In the next charter we take a brief look at some suggested sclution methods for the TSP and their applicability to the buoytender problem.

III. A BRIEF SURVEY OF SOLUTION METHODS FOR THE TSP

In this chapter a brief survey of the general solution methods for the TSP and some of their associated algorithms will be presented. The number of algorithms proposed for solving the TSP is extensive. To present all of the available algorithms or to discuss them in detail is worthy of a dissertation in itself. This survey of solution methods is made because a combination of a couple of the methods will be used to solve the buoytender problem. This survey will be brief in nature presenting the general solution methods and briefly describing some of the more popular of published algorithms which fall under these solution method headings.

The solution methods may be classified under four general headings. These headings are Tour Building, Subtour Elimination, The One-Tree Formulation, and Tour Improvement. These headings describe how the traveling salesman problem is approached.

A. TOUR BUILDING

Tour Building solution methods to construct a tour using the n nodes and the available arcs in the problem. Tour Building contains both exact and approximate algorithms. Some of the approaches taken by Tour Building Algorithms to solve the TSP are dynamic programming, 'branch and bound' methods, and tour construction heuristics.

Three excellent articles [Ref. 1,3,4] have been published which describe the TSP, discuss general solution methods, and review many of the algorithms which sclve it. These three articles review the research conducted on the TSP from the mid 1950's up to 1983.

Held and Karp [Ref. 5] present a dynamic programming approach which is suitable for small problems and can be modified to an approximate method for larger problems. The dynamic programming approach rapidly consumes available memory as the size of the problem increases. Due to memory limitations, the algorithm Held and Karp presented is exact only up to 13 cities. They also present a dynamic programming method which approximates the solution for larger prob-The approximation is done by partitioning the large problem into several smaller problems or subproblems, each subproblem containing 13 or fewer nodes. Each partitioned subproblem is solved for its optimal tour resulting in a set of subtours for the original problem. The subtours are then broken and linked to each other to provide a tour for the original large problem.

Little et al. [Ref. 6] present a branch and The algorithm branches on whether a particular arc is included or excluded from the tour being constructed. This tranching partitions the solution set containing all tours into subsets. Each arc in the total graph can be associated with some subset of tours. and the idea is to find the subset containing the optimal tour. For each subset of tours, a lower bound is computed for the tours within the subset. The tour is constructed as arcs with favorable lower bounds are included in the tour. The algorithm continues branching and computing lower bounds until a subset is found which contains one tour and this tour's distance is less than or equal to the lower bounds of the other subsets of tours. This tour will be the optimal solution to the TSP. The memory required for this technique may te extensive and computing time grows exponentially with the size cf the problem.

Many tour construction heuristics have been proposed. Rosenkrantz, Stearns, and Lewis [Ref. 2] and Golden et al.

[Ref. 4] review several of these heuristics. The two most popular forms of these heuristics are the nearest neighbor algorithm and the insertion techniques.

The nearest neighbor algorithm operates exactly as its name implies. A starting node is chosen and its nearest neighboring node is selected as the next node in the tour being constructed. The algorithm then iteratively selects a node not yet in the tour which is nearest to the previously selected node to enter the tour. This selection continues until all nodes have been selected and then the last node selected is connected to the first node to complete the tour.

The insertion techniques include the nearest, farthest, cheapest, and arbitrary insertion algorithms. Insertion techniques begin with a starting node and select the nearest node to create a subtour. The algorithm iterates selecting a node which is nearest to or farthest from any other node in the subtour, or arbitrarily selects the next node to enter the tour. This selected node is then inserted into the subtour wherever it provides the least increase (or in the case of the farthest, the greatest increase) in distance These heuristics generally provide a in the new subtour. suboptimal solution to the TSP, but they have the advantage cf being very simple to implement and have very fast solution times with minimal memory consumption.

B. SUBTOUR ELIMINATION

The Subtour Elimination solution method [Ref. 1] attacks the TSP by solving an n by n assignment problem with the added constraints that the final solution must contain a cycle and the cycle cannot be of size n-1 or less. The number of integer constraints necessary to eliminate all subtours or cycles of size n-1 or less is enormous.

Therefore, the initial solution is generally found using a 'relaxed' form of the assignment problem. The relaxed assignment problem cmits the subtour constraints in its formulation.

Mathematically stated the relaxed assignment problem is

Min
$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_{i,j} d_{i,j}$$

S.T. $\sum_{j=1}^{n} x_{i,j} = 1$ $i=1,2,...,n$
 $\sum_{i=1}^{n} x_{i,j} = 1$ $j=1,2,...,n$
where $x_{i,j} = 0$ or $x_{i,j} = 1$

The solution to the relaxed assignment problem provides the initial lower bound on the optimal tour length. optimal assignment sclution is a tour then it is an optimal solution to the TSP. If the assignment solution is not a tour then there exist subtours which must be removed until a single tour exists. If a subtour exists with k arcs in the subtour, then there are k possible subproblems to which the problem may branch. These k subproblems each have an additional constraint, the constraint for subproblem (i=1,...,k) being the exclusion of arc i from the problem to eliminate the subtour. Instead of branching into k subproblems the approach is to branch into two subproblems. An arc is selected from the subtour and the exclusion of this arc becomes a new constraint in one of the subproblems while inclusion of the arc in the tour becomes a new constraint the alternate subproblem. The modified assignment problem is solved again for each of these new constraints and provides the lower bound for its respective branch. this way only the necessary subtour elimination constraints are added as needed to the assignment problem, rather than attempting to add all possible subtour constraints to the initial assignment problem. Ideally the branch solution subset with the lowest bound is then selected to be tested

to see if it is a tour, and, if it is not, the problem tranches again on the lowest bound. This branching continues until all subtours have been eliminated and a tour is found which has a distance less than or equal to the lower bounds of the other solution subsets.

There are two lasic methods for searching created by the branch and bound process. One method is a breadth-first search. As the program is branching computing bounds, a tree is created with leaves or terminal nodes. A breadth-first search requires storage for solutions of each leaf and a search through these leaves for the next subproblem on which to branch, This requires extensive, usually exponential, storage. Branching at the lowest bound requires a breadth-first search. The second method for searching the tree is a depth-first search. depth-first search is a more localized approach to handling The derth-first search need only store If during the immediate sclution while searching a branch. search it is found that no further branching is possible from the current node, the process must return to a previous leaf to continue the search for the optimal tour. The depth-first search tacktracks and recreates the previous solution at that leaf. The depth-first search does require exponential storage like the breadth-first search, but it usually requires more computation time.

The subtour elimination method is described as being exact. The subtour elimination method usually requires less computation time than the branch and bound approach of little et al. since the bounds obtained with the assignment relaxation are tighter.

C. THE CHE-TREE FORMULATION

The one-tree formulation was introduced and developed by Held and Karp [Ref. 7,8]. The general method for the solution of the TSP begins with the construction of a minimum-weight 'cne-tree.' If the one-tree is not a tour then an integer linear program may be used to obtain the final solution.

To assist in the description of this solution method several terms need defining. First a tree is defined as a connected graph without cycles. A minimum-weight spanning tree is a tree with minimum total weight on its edges. A one-tree is a tree to which one edge has been added yielding exactly one cycle. A minimum-weight one-tree is a one-tree with minimum total weight on its edges. Finding a minimum-weight one-tree is a relaxed version of a TSP since the solution to a TSP is a minimum-weight one-tree having every vertex of degree two.

In the solution method proposed by Heli and [Ref. 7] the algorithm begins by constructing a minimumweight one-tree. A minimum-weight one-tree can be can be found by first constructing a minimum-weight spanning tree through nodes 2 to n and then adding to the graph the two arcs of least weight from node 1 [Ref. 9]. weight one-tree may also be constructed by constructing a minimum-weight spanning tree through nodes 1 to n and then add to the graph the next minimum-weight edge not yet used from the distance matrix. If the one-tree constructed is a tour (i.e. the one-tree's vertices are all of degree two) then the TSP is solved. Otherwise, the one-tree must be converted into a tour.

Integer linear programming is used to obtain an optimal tour if the one-tree formulation does not satisfy a tour. Held and Karp introduce the concept of a 'gap' function and

then use a special integer linear program to minimize this function. In essence there is a transformation on the onetree variables which allows the integer linear program to search for the optimal tour. Held and Karp suggested two methods for optimizing the integer linear program, an ascent method and a branch and bound method with the ascent method embedded in it. Other improvements to the Held and Karp method were suggested by Hansen, Krarup [Ref. 10] and Houck, Picard, Vemuganti [Ref. 11]. Bazarra and Goode [Ref. 12] did further work on Held and Karp's approach using optimization of a lagrangian dual in lieu of solving integer linear They proposed a branch and bound scheme with subgradient optimization of the dual to transform the one-tree into the optimal tour. The one-tree method as described is exact and may require extensive computation time.

D. TOUR IMPROVEMENT

The Tour Improvement method assumes that an arbitrary tour is available. The method operates by perturbing or exchanging arcs in the tour until a better tour is found. The Tour Improvement method terminates when a better tour cannot be found. In its simplest form, it is possible for this method to check (n-1)!/2 arc exchanges for the symmetric TSP before terminating (i.e., it investigates all available answers).

Dantzig, Fulkerson, and Johnson [Ref. 13,14] proposed a algorithm which starts with an arbitrary tour and uses integer linear programming to improve the tour. The TSP is transformed into an integer linear program and solved using the simplex method. Dantzig, Fulkerson, and Johnson [Ref. 15] mention that the number of constraints needed to characterize the problem is 'astronomical.' Instead of using all the constraints, they begin by using a relaxed version

of the problem and .dd constraints as needed to the integer linear program to maintain feasibility. In their method infeasibility appears as 'loop' or subtour. To continue solving the problem a constraint is added which removes the subtour yet does not eliminate any of the available tours. This algorithm requires the addition of constraints during the solution process to maintain feasibility. Several researchers have suggested improvements on Dantzig, Fulkerson, and Johnson's method for solution of the TSP. [Ref. 16,17,18,19]

Croes [Ref. 20] proposed a heuristic whereby a subset of all possible transformations is tested, a transformation teing the transformation of one tour into another tour. called these transformations 'inversions' because they inverted the sequence of nodes in part of the tour to create a possible improvement. Lin [Ref. 21] later describes Croes 'inversion free tours' as 2-optimal tours and goes on to describe k-optimality where k is some fixed number less than The general idea is to exchange k arcs iteratively in the tour while testing for improvements. Lin and Kernighan [Ref. 22] presents a modified k-opt algorithm where k is not In this algorithm k may vary from 2 to floating k-opt algorithm is also described by Christofides and Eilon. [Ref. 23]

The advantage of the 'inversions' or k-opt solution method is that for a given problem, the memory needed to solve the problem is fixed. There are no constraints which must be added to the problem, and the decision rule used is simple. The k-opt method is exact, but may be terminated before optimality is reached to provide a satisfactory (suboptimal) solution. This generally reduces run time and still provides a good solution to the problem. The k-opt method may also be programmed to require very little user interaction; in the best case the user need only enter the

distance matrix for the nodes desired in the tour. The k-opt algorithm may require extensive computation time, particularly if during solution k is found to be greater than 2 on many of the iterations.

This chapter has presented a general survey of some the solution methods for the TSP and some algorithms associated with these solution methods. The next chapter will discuss which of these solution methods were employed to solve the bouytender problem. The buoytender routing algorithm will then be presented and discussed in detail.

IV. BUOYTENDER ALGORITHM

This chapter will discuss the algorithm with which we propose to solve the buoytender problem. The reasons for the algorithm's selection and some background on its development will be presented, together with a detailed description of how the algorithm addresses the buoytender problem.

A. ALGORITHM SELECTION

The algorithm is a combination of two of the previously mentioned solution methods in that it combines a Building heuristic with a Tour to Tour Improvement heuristic. The concept of combining a Tour Building heuristic with a 2-cft or 3-opt heuristic was proposed by [Ref. 24] as a relatively fast computational Golden et al. solution method which would provide an optimal or optimal tour. Their 'composite' algorithm is the foundation on which the buoytender algorithm was developed.

The heuristics used in the buoytender algorithm are the nearest neighbor algorithm and the k-opt algorithm. The nearest neighbor algorithm is used to construct an initial tour while a version of the k-opt proposed by Lin and Kernighan [Ref. 22] is used to improve this initial tour. The nearest neighbor and k-opt heuristics were selected for use in the buoytender algorithm because very little interactive work is required, the interactive work is simple, no constraints need to be added to maintain feasibility, computational time is reasonable, and the algorithm operates with a fixed amount of memory.

The huoytender algorithm as presented requires very little user interaction. The nearest neighbor algorithm

needs only the entry of the distance matrix for the problem and the selection of a starting node. The k-opt algorithm needs the same distance matrix and an initial tour. The heuristics' decision processes will provide a feasible and possibly optimal solution in a reasonable period of time. The nearest neighbor algorithm was selected to construct the initial tour because it is computationally faster than insertion techniques, and provides a good initial solution to the problem. [Ref. 25]

A critical factor in solving the TSP on a microcomputer is the memory required by the algorithm used. The amount of memory required for solution of a TSP of size n is fixed for the nearest neighbor and k-opt heuristics. With known memory requirements for the heuristics, a program can be developed to fit the Coast Guard Standard Terminal. The nearest neighbor algorithm requires an n by n distance matrix and an n arrary in which to store the tour as it is constructed. The k-cpt algorithm requires the same storage as the nearest neighbor plus a n by n decision matrix, and a 2n array for recording nodes selected for inversion in the tour.

B. PROGRAMMING THE ALGORITHM

The huoytender algorithm was developed as a series of modules which were then programmed as subroutines in the operating program. The algorithm was programmed in FORTRAN for operation on the Coast Guard Standard Terminal. FORTRAN was selected because it is efficient and a majority of programmers are familiar with it.

There are some differences between the heuristics in the literature and the buoytender algorithm. The nearest neighbor algorithm as used by the buoytender algorithm is the same as outlined by Rosenkrantz, Stearns, and Lewis

The k-opt heuristic used in the buoytender algorithm is a modified version of the heuristic presented by Lin and Kernighan [Ref. 22]. There are three major lifferences between the k-crt in the bouytender algorithm and the k-opt suggested by Lin and Kernighan. First, the Lin and Kernighan k-opt heuristic selects all possible nodes which will result in a shorter tour before creating the improved The buoytender algorithm improves the tour by exchanging nodes whenever a favorable selection is found. This improvement-as-you-go procedure results in a simplified selection process for the next set of nodes to be tested. Second, the Lin and Kernighan k-opt heuristic has an added facility for 'limited backtracking' used when a particular exchange gives an improvement of zero. The backtracking procedure searches until a gain greater than zero can be found which improves the tour. In the interest of shortening computational time, the buoytender algorithm does not 'backtrack' when a gain of zero is found, but instead treats gain as no improvement and continues with the next selection. The third major difference between the Lin and Kernighan k-opt heuristic and the buoytender algorithm is related to what Lin and Kernighan call 'reduction.' Their heuristic, after producing several locally optimal tours, checks for arcs which appear in each of the tours. 'good' arcs are not allowed to be broken in further computations for other locally optimal tours thereby reducing the number of links to be checked for improving the tour. While decreasing run time, this procedure requires more memory and is therefore omitted from the buoytender algorithm.

The huoytender algorithm can be divided into three basic operations. First, the algorithm selects a random sample of k nodes, k<n. This random sample of nodes becomes the set of starting nodes for the next major operation in the algorithm, to create k nearest neighbor tours. The third

operation is to use the k nearest neighbor tours as initial tours for the k-opt improvement. The best of the k-opt improvement tours is presented as the solution to the problem.

A further modification was made to the algorithm based on some computational results obtained from use of an early version of the algorithm. During programming of the buoytender algorithm several benchmark statistics were obtained from the program. Two of these statistics were the initial tour distances produced in the nearest neighbor phase, the distances of the final tour solutions produced by the k-opt phase of the algorithm. There seemed to be a relationship between the tour produced by the nearest neighbor phase and the improved tour produce by the k-opt phase. least-squares linear fit of the initail tour distance vs. the final tour distance was done for four data sets. each case the fit produced a positive slope. This implies for a relatively large nearest neighbor tour, the final tour from the k-opt phase will be relatively large. A relatively small nearest neighbor tour will produce a relatively small tour from the k-opt phase. Using this information the buoytender algorithm was further modified to reduce computation time without compromising too much of its ability to produce optimal solutions. For a set of k nodes, k<n, the nearest neighbor algorithm provides k initial tours. The set of k nodes is selected at random from the population of n nodes. Then for a set of L ncdes, L<k, the k-opt algorithm is run to obtain L final tours. The set of L nodes are drawn from the k nodes which produced the L shortest initial tours. The shortest final tour from the L final tours produced by k-opt phase will be the algorithm's solution to problem. The final version of the algorithm has a greatly reduced operating time over the initial version.

C. AIGORITHM DESCRIPTION

In this section, a detailed description of the buoytender algorithm is given. In this description references will be given to the program subroutines so that the reader may associate the portion of the algorithm under discussion with its operational counterpart in the program.

The buoytender algorithm begins by reading a file from mass storage containing the number of aids (nodes) in the problem, the distance matrix, and the names of the aids (subrcutine OBTAIN). A program MATUTL.FOR, which assists the user in creation of this data, can be found after the listing of the buoytender route selection program, ROUTE.FOR.

After obtaining the data the algorithm selects a random sample of nodes, based on the size of n, from which it will create tours (subroutine NODE). The function which selects the size of the random sample is

Sample Size =
$$5 + \lceil 2(\log(n-4)) \rceil$$
.

The function is designed to capture almost the entire population for testing when the problem is small and to capture only a small portion of the population being tested when the problem is large. This produces a sample size which increases the probability of obtaining the global optimum when the problem is small yet produces a sample size which will maintain reasonable computation times when the problem is large.

After determining the sample size, a random sample of nodes is taken. Since there is no random number generator for FORTRAN installed on the Coast Guard Standard Terminal, a pseudo-random number generator was added to program (function RANDOM). The coding for the generator comes from Wichmann and Hill [Ref. 26]: it is described by its

authors as an efficient psuedo-random number generator having a cycle length of 2.78 x 10¹³. Nodes are sampled at random without replacement from the integer population of 1 to n for use as starting nodes for the nearest neighbor phase.

The algorithm then moves into the nearest neighbor phase (subroutine NBR). A tour is created for each node in the random sample, with the node from the sample as the starting node of the tour. The algorithm selects the nearest node to the starting node for the next entry in the tour construction. The algorithm then iterates selecting a node not yet in the tour which is nearest to the previously selected node as the next entry in the tour. This process continues until all nodes have been selected, at which time the last node is connected to the starting node to complete the tour, and the tour distance is computed. After all the initial tours have been constructed they are ordered by their total distances from shortest to longest. The five shortest tours are retained for possible improvement in the k-opt phase.

A decision matrix is created which designates which distance arcs in the distance matrix are presently members of an initial tour (subroutine MARKD). The algorithm then proceeds to step through each node of the tour testing it for k-optimality.

The tour is 'prepared' at each node to be checked for k-optimality (subroutine TRPREP). This process simplifies some of the later operations. Here, it is helpful to consider the tour as a circle with n postions on its circumference (see Figure 4.1), and to let positions on the circular tour be denoted by p(i) where i=1,...,n. A node in the tour is represented as a position on the circle. TRPREP places the starting node in p(1) and places the remaining nodes in the tour at p(2) through p(n) where (i) is the node's position in the tour.

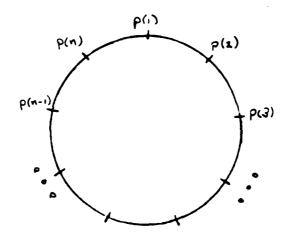


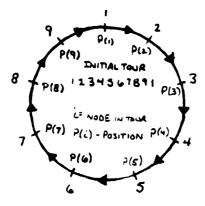
Figure 4.1 Circular Representation of a Tour

Since floating k-cpt algorithm is in essence a series of 2-opt iterations a description of the simpler 2-opt principle will be presented first. Figure 4.2 shows the 2-opt principle in graphic detail. In Figure 4.2 (A), the tour is represented in circular form. The tour is now perturbed to try and find an improvement. To perturb the tour in the 2-opt procedure, two arcs will be broken in the existing tour and two new arcs will be selected as incoming to form a new tour. This provides a gain function which can be used to check for improvement. The gain function is

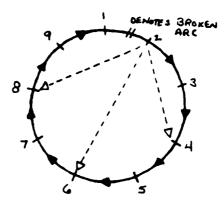
 $G = \sum_{i=1}^{n} (broken arcs) - \sum_{i=1}^{n} (incoming arcs)$.

If G is positive this set of arc exchanges will improve the tour.

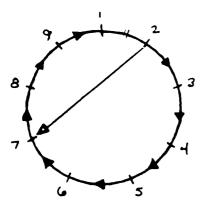
To perturb the tour, one of the arcs incident to node being checked for 2-optimality must be broken or removed from the tour. In the buoytender algorithm the node being checked for k-optimality will always be in p(1) of the circular tour and the arc being broken will always be between p(1) and p(2) of the circular tour. An incoming arc must then be selected to replace the broken arc. This incoming arc must be selected from those arcs with a starting node which is the same node as the end node of the



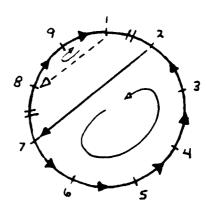
(A) Circular Tour



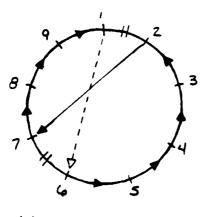
(B) Search for Incoming Arc



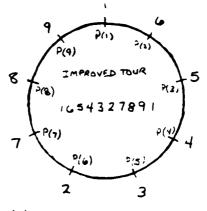
(C) Incoming Arc Selected



(D) Wrong Incident Arc Broken



(E) Feasible Arc Exchange



(F) Circular Tour

Figure 4.2 Graphic Description of 2-opt

previously broken arc. Figure 4.2 (B) shows some of the available selections for the _ucoming arc. The incoming arc

is in general the shortest available arc, but may be any of the available arcs which maximize G. This subtour must be broken and another incoming arc must be selected so as to reconstitute a tour. Therefore, one of the tour's arcs incident to the end node of the first incoming arc must be The end node in Figure 4.2 (C) is at p(7). is only one of two possible tour arcs which may be broken to allow the incoming arc to become part of the tour. shows the selection of the wrong incident tour arc T he breaking of this arc and the subsequent reconnection to the node being checked for 2-optimality does not result in a tour. Instead. two subtours are created giving an infeasible situation. Figure 4.2 (E) treaking of the proper incident arc and subsequent reconnection to the original node to create the 'improved tour'. Also notice that some of the arc directions in the original tour must be reversed to provide a consistent direction of movement through the tour. Reversing these arc directions is the same as inverting a sequence of nodes in The result is shown in Figure 4.2 (F) as a new 'improved' circular tour.

The 2-opt procedure is carried out using each node in the tour as the starting node checked for 2-optimality. One interation of the 2-opt procedure through all n nodes will produce a tour which is 2-optimal.

The k-opt algorithm is very similar to the 2-opt procedure described above. Instead of just reconnecting the tour as the 2-opt procedure does (p(6) to p(1)), the k-opt checks to see if another incoming arc, starting at the end node of the last broken arc, can be selected to further improve the tour (Figure 4.3). The procedure interates until no further improvement can be found. A k-opt, using an example with k equal to four, might look like Figure 4.4 The floating k-opt algorithm has the ability to vary k during the k-opt

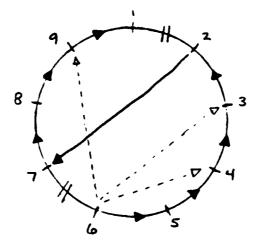


Figure 4.3 K-opt Selecting Next Incoming Arc

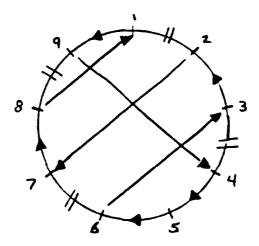


Figure 4.4 Example of 4-opt

solution process. This is more desirable than fixing k in the solution process. For example, if k were fixed at four, the algorithm can only check four arc exchanges for improvement but the problem may need a five arc exchange to produce the optimal solution. By allowing the k-opt to vary k, the probability of the algorithm finding the optimal solution is increased. The floating k-opt is bounded between 2-optimality and n-optimality. The k-opt cannot be greater than n-optimal because an n-opt exchange creates the optimal tour.

The k-opt procedure in the buoytender algorithm operates by checking each node in a tour for k-optimality. The algorithm, using subroutine TRPREP, creates the circular tour for the node being checked for k-optimality. The circular tour is prepared so that the initial arc being broken will always be between p(1) and p(2). The algorithm checks the circular tour at the starting node for k-optimality. If no improvement can be made breaking the first arc between p(1) and p(2) then the circular tour's direction is reversed (subroutine REVTOR) so that the alternate incident arc is placed between p(1) and p(2) of the circular tour and this tour is checked for k-optimality.

The breaking of the arc between p(1) and p(2) requires an incoming arc to replace it (subroutine SELCTY). algorithm selects the five shortest available arcs which originate at position two of the circular tour and terminate at another position on the circular tour. The arc from p(2) to p(3) is not available since it is already in the tour, and the arc from p(2) to p(1) is not available since it is the arc just broken to improve the tour. For each of the selected incoming arcs another arc must be broken to eliminate a subtour (as in the 2-opt procedure). It is then necessary to reconnect the arcs to reconstitute the tour. This results, for each of the selected arcs, a set of two incoming arcs and two outgoing arcs. Using the previously defined G, each of the five sets of arcs is checked to see which maximizes G. If G>0 then the exchange or inversion is made (subroutine ADJTCR), otherwise no exchange is made.

If an exchange is made the algorithm goes back and searches for a new incoming arc from the end point of the last broken arc. Subroutine ADJTOR has inverted the tour so that the last arc used to reconnect the tour is now between p(1) and p(2) of the circular tour. Subroutine SELCTY is again used to select the next best available incoming arc as

described above. The algorithm interates through selecting an incoming arc and inverting the tour as long as improvement, i.e. positive gain, can be found.

When improvement can no longer be found the direction of the tour is reversed at the last node where improvement was found, and the tour is tested again for possible further improvement. When no further improvement can be found and the reversed tour has also been tested, the algorithm then increments to the next node in the tour and starts the k-opt procedure again testing this node for k-optimality. Each node in the tour is used as a starting node in the k-opt procedure. When all nodes in the tour have been tested for k-optimality the k-opt phase ends and the resulting tour is stored.

After each of the five shortest initial tours have been tested for k-optimality, the shortest of the improved k-opt tours is selected as the solution to the buoytender problem. Figure 4.5 is a flowchart of the buoytender algorithm.

The next chapter discusses the results of some test problems solved by the algorithm and makes concluding remarks about the algorithm.

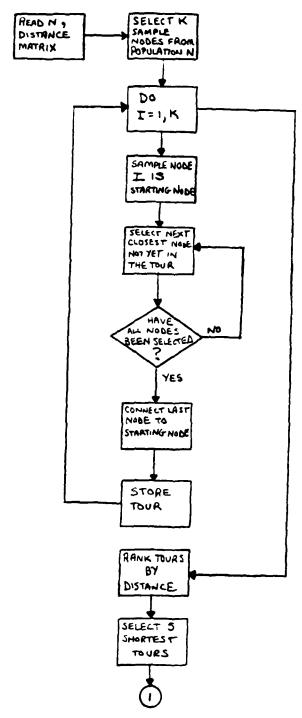


Figure 4.5 Algorithm Flowchart Section 1

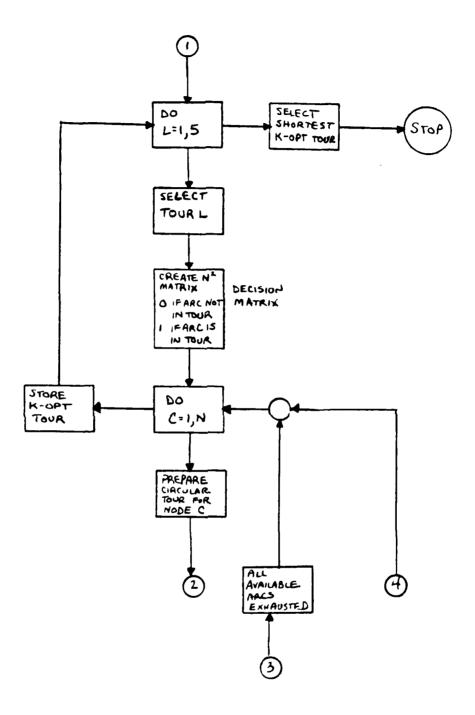


Figure 4.6 Algorithm Flowchart Section 2

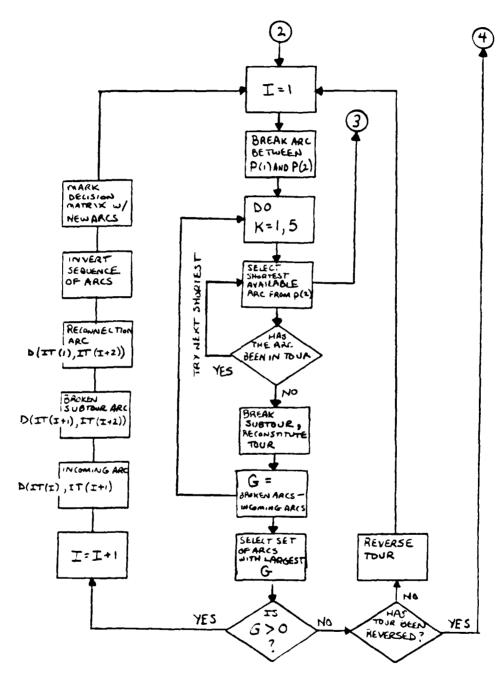


Figure 4.7 Algorithm Flowchart Section 3

V. PROGEAM RESULTS AND CONCLUSIONS

This chapter will discuss some results obtained from using the huoytender route selection program, including results from an actual problem from the U.S. Coast Guard Cutter BIACKHAW. All program results were obtained using the Coast Guard C3 Standard Terminal. The chapter will conclude with some observations regarding the bucytender problem, the algorithm, and the test results.

A. FROGRAM RESULTS

To program and test the buoytender algorithm it was necessary to obtain problems with known optimal solutions. Initially, a very simple and small problem with nine nodes was created. This test problem's optimum tour is the most obvious tour obtained by inspection of the problem since the are arranged in an almost circular Connecting these nodes following this nearly circular path results in the optimum solution. Several alternate tours were checked by hand computation and were found to be suboptimal. Therefore, the near circular path for this test problem is assumed to be optimal. When the program was tested with this test problem, the program produced this hypothesised optimum solution with a computation time of 13 seconds.

With the knowledge that the program appeared to be operating correctly on the initial test problem, further testing was necessary to check the program's ability to solve other problems. Since the huoytender route selection problem is a symmetric case of the TSP, the literature was searched for test problems. Three articles [Ref. 5, 20, 27] yielded eight

symmetric TSP's of various sizes. These eight literature problems ranged in size from 5 to 57 nodes with known optimal answers. The program, when tested with these problems, produced solutions which ranged from the optimal to within 3.5 percent of the optimal distance.

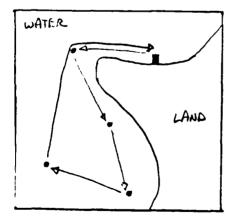
Another problem was created by the random selection of 70 pcints on a Euclidean plane. A distance matrix was computed for these 70 nodes and this problem was solved by the program. The problem's optimal solution is not known, but the problem was run to ensure the program could handle the maximum problem for which it was designed, and to produce a sample solution time for this maximum sized problem.

With evidence that the program was solving test problems, a further test was conducted using an actual buoytender routing problem, with data provided by the U.S. Coast Guard Cutter BLACKHAW (WLB-390). homeported in Fransisco. The aids to navigation run data was for a trip scheduled from 9 April 1984 to 16 April 1984. experience and historical data this vessel has available from servicing these aids for many years, it is hypothesized that the route which was scheduled for this ATON run would be optimal or nearly optimal. The planned route for the ATON run had a distance of 455 nautical miles. When this problem was entered and solved by the buoytender program, the sclution was also 455 nautical miles. Although distances were the same, the scheduled route and the program route differed slightly in their sequence for visiting the This difference may be attributed to the fact that, as stated in Chapter II, the triangle inequality may be required to be a strict equality on some of the arcs. difference is possible since the buoytender may, due to the strict equality on some arcs, he required to 'pass' a previously serviced aid to complete the route. Figure 5.1 shows

a five node example of this situation. This suggests there may be several sequences of nodes which will provide an optimal solution to the buoytender problem.

TABLE 1
Tours for BLACKHAW Problem

Route as Scheduled <u>by the Buoytender</u>		Route Produced by the Program
10 WESTDAHL R LEE1 11 FT SANLUIS LWE3 12 LANSING R B 13 FT BUCHON LWE2 14 MORRO BY LBB1 15 ESTERO BY LWE	121116 5 430 21	ESTERO BY LWB ESTERO GB10E MORRO BY LBB1 PT BUCHON LWB2 WESTDAHL R LBB1 SOUZA R LGB



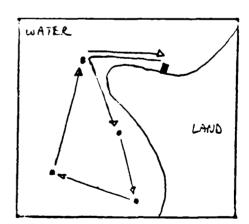


Figure 5.1 Multiple Optimal Solutions

TABLE 2
Results of Test Problems

Problem & (Ref.)	<u>Size</u>	Sclution Time(Sec)	Tour Dis <u>Program</u>	t. (Miles) Optimal	Solution <u>Cuality*</u>
Karg(27) Test(auth.) Barachet(27) BLACKHAW Croes(20) HeldKarr(5) Karg(27) Dantzig(27) HeldKarr(5) Karg(27) Max(auth.)	59122234457	517 17 17 552 722 188 193 494	148 22.8 95 378 455 254 1711 10929 705 11847 13367 6791.07	148 22.8 378 455 246 1711 1086 11997 11295 unk.	100% 100% ** 100% ** 100% 100. 62% 100. 86% 103. 17%

100% is optimal: 103.28% is 3.28% greater than optimal.
 The distance is hypothesized to be optimal.

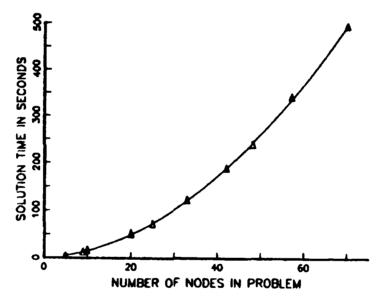


Figure 5.2 Least Squares Fit of Size vs. Solution Time

Table 2 presents a summary of the eleven test problems showing problem sizes, solution times, and the quality of the solutions produced by the program. All of the run times are reasonable by the criteria stated in Chapter II. The largest problem, 70 nodes, had a solution time of 494 seconds, or roughly a little more than eight minutes.

Computation times for the eleven problems were plotted against problem size, as shown in Figure 5.2 A least-squares polynomial fit of the data points is

Time = $.584 + .62n + .0914n^2 + .00000994n^3$ and this curve is also shown in the figure.

E. CCNCIUSIONS

The buoytender route selection algorithm provides another useful way of scheduling ATON routes. As a tool for the operator to assist in planning the route to be taken, the program should produce optimal or near optimal solutions for problems up to a size of 70 aids. The user may accept this route as is or may desire to modify the route based on circumstances requiring human judgement. Appendix A is a user's guide for the matrix utility and buoytender route selection programs.

The buoytender route selection algorithm, with the nearest neighbor and k-opt heuristics, provides a quick and satisfactory solution to a symmetric traveling salesman problem on problems up to a size of 70 nodes. The problem may have several routes with different sequences for visiting the aids, but the routes may all have the same distance.

It is hoped that this algorithm will be of use to the fleet cf U.S. Coast Guard buoytenders. As a decision aid it may help the operators of buoytenders obtain optimal or nearly optimal routes to service their aids, possibly saving time and fuel.

APPENDIX A USER'S GUIDE TO PROGRAMS

The intention of this appendix is to provide instructions for the use of the buoytender route selection program and utility programs. It should be noted that pricr to using the huoytender route selection program, the user will need to use the matrix utilities program listed at the end of this appendix to prepare a data file.

The buoytender route selection program and the matrix utilities program are written in Convergent FORTRAN. Convergent FORTRAN is FORTRAN 77 compatible. The programs should be typed in as presented and linked to the operating system. The Convergent FORTRAN manual has instructions concerning compiling and linking. Recommended names for the run files are ROUTE.RUN and MATUTL.RUN. Both programs will operate on either a IWS or AWS station with 264K memory or more. The programs run about three time slower on an AWS terminal than on an ITS terminal.

The user will be required to enter text and numerics into the programs. All numeric entries are to be integer entries (no decimal point) with the exception of the distance entries. All distance entries require a decimal point entry for proper input.

MATUTL.RUN is capable of entering problems up to a size of 100 aids. ROUTE.RUN is designed to handle problems up to a size of 70 aids. MATUTL.RUN allows the entry of a large matrix of aids and includes a utility which allows the user to select a subset of this large matrix for use in the program ROUTE.RUN. This allows the user to type in one large matrix and then create smaller matrices from the larger as needed. This feature will preclude the user from

typing in a distance matrix every time he desires to run a problem.

A. USE CF MATUTL RUN

The matrix utilities program is designed for creating and editing data files which will be used in the buoytender route selection program. A data file will contain the size of the problem n, the distance matrix for the problem, and the names of the aids to be visited.

The size of the problem, n, will be the number of aids to be visited plus one. The plus one is for the port from which the buoytender will start and end the ATON run.

To create the distance matrix some extensive plotting is required. Most buoytenders have a set of tracklines they regularly follow to visit their aids. In most cases these existing tracklines will be sufficient to use in the distance matrix. To create a distance matrix for entry into a data file it is recommended the user use the format shown in Figure A.1. Distances should be computed in nautical miles and may range from .01 nm to 9999.99 nm. Figure A.1 is an example of a problem of size nine.

Since the problem is symmetric, the user need only calculate and enter the distances as shown in Figure A.1. The matrix utilities will automatically complete the rest of the matrix entries. The distance you should compute should be the shortest possible distance from the row entry to the column entry. Using the example in Figure A.1 the entry in row 3, column 6 is the shortest possible trackline from aid A to aid E. Once the entries for the distance matrix have been prepared the user is ready to use MATUTL.RUN.

MATUIL.RUN has some simple menus and some descriptive comments to help the user. The menus require a numeric response for the selection of an option. The first menu presents the utilities available to the user, Figure A.2.

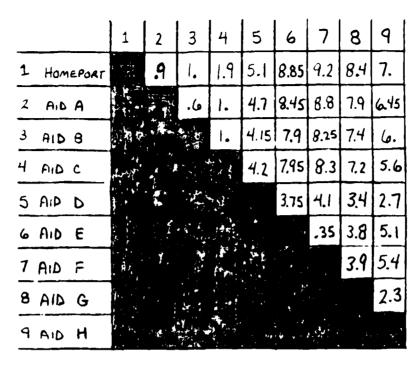


Figure A.1 Sample Distance Matrix

```
- create a distance matrix
- extract a small matrix from a larger matrix
- correct a previously stored matrix
- create a hardcopy of a distance matrix
- list names with their row number in the matrix
- exit this program
```

What is your selection?

MATUTL.RUN Main Menu Figure 1.2

1. Create a Distance Matrix

This utility is for the initial entry of a distance The user will be asked for the matrix into a data file. and the size of the problem, the homeport and aids names,

distances. The size of the problem is the number of aids plus one, for homeport. The utility will next ask for names of homeport and the aids to be visited. These names will be associated with a row and column number of the distance matrix. The names are text entries up to a maximum of 15 characters.

The utility will next ask for distances. It will ask for the distance from a row number to a column number. If the user sets up the distance matrix as shown in Figure A.1 he will be able to then just read each row across its columns while entering the data. These distances must be The distances should be nautical miles and may range from .01 nm to 9999.99 nm. Since these entries are real numbers there must be a decimal point in each If, for example, the user needs to enter nautical miles. he should enter it as 2. <return>. careful with these entries: if a decimal point is not enter∈d the distance will be off by a factor of 100. quickly and easily entering data the numeric keypad on the keyboard may be used and <next> may be used in place of <return> to enter the data. If an error is made typing in numbers and <return> or <next> have not yet been pressed, then the error may be corrected by using the backspace key and retyping the number. If an error is made and <return> or <next> have already been pressed, then the user should note the row and column number of the error and use the correction utility to correct the entry.

When the user finishes typing all entries the program will prompt for a filename in which the data will be stored. This name may be up to 15 characters long. If for some reason the user needs to end the terminal session before the entire matrix is entered, data entry may be stopped by typing in a negative distance. The user will then be asked for a filename for the partial data file. To

complete the partial data file the user must use the correction utility.

2. <u>Extract a Matrix</u>

This utility is designed so that the user may choose a subset of aids from a larger matrix. The utility will create the proper data file for a subset of aids.

The user should have an idea of all possible aids which may be visited on a general buoytender run (i.e. the spring run, the north run, the inside run, etc.). The user should create a distance matrix using the <u>create</u> utility of all aids which could possibly be visited on a general run. Usually cnly some portion of these aids will be visited on an actual run (for example on a particular spring run only 90% of the aids which might be visited on this run need to be visited). In this case the user creates a matrix of all aids which might be visited on a run and then each time this run is scheduled the user can then use the <u>extract</u> utility to quickly and easily create a data file for those aids which need to be visited.

Before using the <u>extract</u> utility it is suggested the user first use the <u>list</u> utility on the larger or initial data file. This will provide the user a list of aid names in the data file and present them with their corresponding row number.

The <u>extract</u> utility will prompt the user for the name of the data file containing the general run's aids. The user is then asked for the size of the new problem whereby the user enters some m, m less than n in the initial data file. The utility then asks the user m times for row numbers from the initial matrix. After m row numbers have been entered the utility creates the new data file and askes the user for a filename for this new data file.

3. Correct a Previously Stored Matrix

This utility is to correct entries in a distance matrix or to complete a partially entered data file. The utility has a menu which askes if the user wishes to make single entry or sequential entry corrections. The single entry mode asks the user for the row number and column number of the entry desired to be corrected. The user then is asked for the new distance and is returned to the correction menu where he may select to make another correction or exit the utility.

The sequential corrections are similar to the entry of distances in the create utility. The user is asked for the starting row number and starting column number where The utility asks for the corrections are to begin. remaining column entries on the starting row and then will proceed to the next row until entries are terminated. utility automatically terminates at the end of the matrix or the user may terminate the sequential mode early by entering If a negative number is entered it will a negative number. Terminating the sequennot appear in the distance matix. tial mode returns the user to the corrections menu for another correction or to exit the correction utility.

Upon exiting the <u>correction</u> utility the user is asked for a filename for the corrected file. This filename may be up to 15 characters long.

4. Frepare a Distance Matrix for Hardcopy

This utility is for preparing a hardcopy of a distance matrix from a stored data file. The data files are saved as sequential files, (i.e., the data files are a string of numbers). This utility will construct a file which will resemble a matrix. Due to the width and length limitations of the printer and paper, the matrix is printed

in sections which will have to be cut and pasted together to display the whole matrix. A hard copy of the whole matrix will provide the user with a means of checking to see if data entries are correct. If an entry error is found, the matrix also helps the user to locate the row and column number of the error.

The utility asks the user for the name of the data file from which a distance matrix is needed. The utility creates the hardcopy file and then prompts the user to obtain a hardcopy by using the FORMAT command to print out file MATERI.DATA.

5. List Aid Names

This utility provides a hardcopy of the aid names from a data file. This may be useful to the user when using the <u>extract</u> utility. The user is asked for the name of the data file from which the list shall be made. The utility then prepares a file containing the row numbers and their associated aid names. The user is prompted to use the FORMAT command to obtain a printout of LIST.DATA.

These matrix utilities are for assisting the user in creating and manipulating data files and distance matrices. A correct data file is necessary for the proper operation of the buoytender route selection program, ROUTE.RUN.

B. USE CF ROUTE.RUN

The user of ROUTE.RUN is very simple once a data file has been created. The user runs the ROUTE.RUN program and he is prompted for the name of a data file. The user is then asked for the name of an output file for the results. The program obtains the size of the problem from the data file and will compute an approximate solution time. This approximate solution time is based on results from some test

problems solved by the program and it assumes the user has version 8.01 of the operating system. If another version of the operating system is used the solution time may differ.

After displaying the approximate solution time the program reads the rest of the data file and proceeds to do the necessary computations. The user, depending on the size of the problem, will have just enough time to get a cup of coffee before obtaining the solution. The solution will be displayed on the screen and will also be placed in the file previously named for results, from which a hardcopy may be made. The user now has a possible route to take for that particular aids to navigation run.

APPENDIX B SAMPLE OUTPUT OF EUGYTENDER ROUTE SELECTION PROGRAM

The solution for data file BLACKHAW.MAT Total tour distance = 455.00 The tour is as follows

- 1 YB ISLAND
- 20 MOSS LBB MLA
- 19 MONTRY BY LB B
- 18 MONTRY HER MB
- 17 PT PINOS LWB2
- 16 PT CYPRESS LGB6
- 5 P BLANCAS LWB4A
- 6 SAN SIMEON LBB1
- 7 VON HELM R LGB4
- 15 ESTERO BY LWB
- 8 ESTERO GB10E
- 14 MORRO EY LBB1
- 13 PT BUCHON LWB2
- 10 WESTDAHL R LBB1
- 9 SOUZA R LGB
- 12 LANSING R B
- 11 PT SANLUIS R B
- 4 SANTA CRUZ MB
- 3 AN ISLAND LWB8
- 2 MONTARA LWB10A
- 1 YB ISLAND

APPENDIX C

LISTING OF BUCYTENDER ROUTE SELECTION PROGRAM

```
MAIN PROGRAM *** BUOYTENDER ROUTE SELECTION ***
C
        IMPLICIT CHAPACTER*15 (A)
       REAL G, TDIS, TORDIS, DIS (15)
       INTEGER I, J, L, C, P, NUM, NOD, SAM (15)
       CHARACTER*15 ENAME, ONAME
       COMMON /NAME/ ANAME(70)
       COMMON /WALK/ N. ITOUR (71)
       COMMON / TDST/ TD(73,73)
       COMMON /INVT/ IT(147)
       COMMON /DIST/ D(70,70)
       COMMON /SAMT/ ISTCR(15,71)
        COMMON /TMTR/ ITPTOR(142)
        CALL OBTAIN (FNAME, ONAME, N)
        OPEN(3, FILE=ONAME, STATUS='NEW', FORM='FORMATTED')
       TORDIS=99999.99
       CALL NODE (N, NUM, SAM)
       CALL MBR(NUM, SAM, DIS)
       DO \bar{7} L=1,5
           DC 8 J=1,N+1
              ITOUP (J) = ISTOR (L, J)
           CONTINUE
8
           CALL MARKD()
           DO 100 C=1, M
              CALL TRPPEP(C)
              R = 0
120
              I = 1
              IT(1) = ITOUR(1)
              IT(2) = ITOUP(2)
130
              CALL SELCTY (I, C, G, NOD)
              I = I + 1
              IF(G.LE.(0.0001)) GOTOIIO
              CALL ADJTOR (I, P, NOD)
              GOTC130
110
              IF(P.EQ.1)GOTO103
              CALL PEVTOR(R)
              GOTO127
130
           TDIS=0.0
           DO 140 J=1, N
              TDIS=TDIS+D(ITOUR(J),ITOUR(J+1))
140
           CONTINUE
           IF (TDIS.GF.TOPDIS) GOTO7
           TOPDIS=TDIS
           DO 6 J=1,N+1
              ISTOP (1,J) = I TOUR(J)
           CONTINUE
        COMMITTUE
```

```
DO 28 I=1,N+1
           ITOUR(I) = ISTOR(1, I)
       CONTINUE
       CALL TPPREP(1)
       WPITE(*,20) ENAME
       WRITE(3,29) ENAME
       WRITE (*,30) TOPDIS
       WPITE(3,30) TOPDIS
       WRITE(*,31)
       WRITE (3, 31)
       WRITE(*,32)(ITCUR(I), AMAME(ITCUR(I)), I=1, M+1)
       WRITE (3,32) (ITCUP (I), AMAME (ITCUP (I)), I=1, N+1)
       CLOSE(3)
       STOP
2 🤈
       FORMAT(IX, 'The solution for data file ', A)
       FORMAT(1X,'Total tour distance = ',F8.2)
37
31
       FORMAT(1%, 'The tour is as follows:')
32
       FORMAT(1X, I3, 5X, A)
       END
       SUBFOUTINE OPTAIN (FNAME, ONAME, N)
         IMPLICIT CHAPACTER*15 (A)
         INTEGER N
         REAL TIME
         CHAFACTER*15 FNAME, ONAME
         COMMON /NAME/ ANAME (70)
         COMMON /DIST/ D (70,70)
         APITE(*,6)
         PEAD(*,7) ENAME
         WPITE(*,0)
         READ(*,7) CNAME
         OPEN(4, FILE=FNAME, FORM='FORMATTED')
         PEAD(4, 0, FFP=14) M
         TIME=.504+.52*N+.0914*N**2+.00000004*N***
         APITE(*,17) TIME
         7RITE(*,11)
         PEAD(4,12, PPP=14) ((D(I,J), J=1, 11), I=1, 11)
         FTAD(4,7,DPP=14) (AMAME(I), I=1,Y)
         CLOSE (4)
       FFIUPM
14
         URITE (*, 15)
         CLOSE (4)
       STOP
         FORMAT(IX, Under What file name is the distance matrix? The
         FORMAT(A)
         FOR 'AT(IX,'File name for the results? 'N)
```

```
FORMAT(I3)
          FORMAT(1X//1X,'It will take approximately ',F6.2,' seconds')
1 '
          FORMAT(1X,'to solve this problem. Please wait.'///) FORMAT(F7.2)
11
12
15
          FORMAT(1X, 'Frror upon load')
        END
C
        SUBROUTINE NODE (N, NUM, SAM)
          INTECER N,J,K,ITFMP, NUM, SAM(15)
          COMMON / RAND/ IX, IY, IZ
          IX=15263
          IY=25460
          17 = 30179
          NUM=5+NINT(2.0*ALOG(FLCAT(N-4)))
          DC 2 J=1, NUM
ITEMP=NINT(PANDOM(1.7)*FLOAT(N))
3
              IF(ITEMP.EQ. ")GCTO3
              F = 0
              DO 4 K=1, NUM
                  IF (ITEMP. FQ. SAM(K)) F=1
              CONTINUE
              IF(F.EQ.1)COTOR
              SAM(J)=ITEMP
          CONTINUE
        RETURN
        END
        SUBFOUTINE MEP (NUM, SAM, DIS)
           INTECER NUM, I, J, K, INCOE, INTERP, C, SYM(15), NOFY
           REAL TEMPO, DIS (15)
           COMMON /WALK/ N, ITOUR (71)
           COMMON /DIST/ D(70,70)
           COMMON (SAMT/ ISTOR(15,71)
COMMON (TDST/ TD(77,71)
           DO 10 L=1, NTY
           INODE = SAM(L)
           DO 20 I=1, N
              DO 23 J=1,4
                  TD(I,J) = O(I,J)
23
              TD(I, INCDE) = 9999.93
           CONTINUE
2.2
           ISTOR(L,1) = INODE
           DIS(L)=7.3
           00 21 J=1, N=1
ITEMP=1
```

```
TFMPD=TD(ISTOR(L,I),ITEMP)
             DO 22 J=2.N
                IF (TD (ISTOR(L, I), J) .CF. TEMPD) GOTO22
                ITEMP=J
                TEMPD=TD(ISTOP(L,I),J)
22
             CONTINUE
             ISTOR(L, I+1) = ITEMP
             DIS(L)=DIS(L)+TEMPD
             DC 23 K=1,N
                TD(K, ITEMP) = 9099.09
23
             CONTINUE
          CONTINUE
21
          ISTOR(L,N+1) = ISTOR(L,1)
         DIS(L)=DIS(L)+D(ISTOP(L,N),ISTOR(L,1))
10
          CONTINUE
205
         NOEX=1
         DO 210 I=1, MUM-1
             IF(DIS(I).EQ.DIS(I+1))DIS(I+1)=99999.99
             IF(DIS(I), LF, DIS(I+1))GOTO210
             MOEX=0
             TIMPD=DIS(I+1)
             DIS(I+1) = DIS(I)
             DIS(I)=IFMPD
             DO 223 J=1,N+1
                ITE"P=ISTOP(I+1,J)
                ISTCR(I+1,J) = ISTOP(I,J)
                ISTOR(I,J) = ITFMP
             CONTINUE
220
210
          CONTINUE
          IF (NOEX.NF.1) GOTO275
       FETURN
       END
       SUBROUTINE TPPREP(C)
          INTEGER C,J,K
          COMMON /WALK/ N, ITCUE (71)
          COMMON /TMTP/ ITPTOR (141)
          J = 0
          DO 47 8=1,5
             ITPTOF (K) = ITOUR (K)
             ITPTOP (K+N) = ITOUP(K)
          IF(ITETOR(E).FQ.C) J=K
4.7
          DC 41 P=1,1
             ITOUR (E) = ITOTOR (J+"-1)
41
```

```
ITOUR (N+1) = ITOUR(1)
        RETURN
         END
        SUBROUTINE MARKE ()
           INTEGER J, K
           COMMON /WALK/ N, ITCUR (71)
           COMMON /TEST/ TE (70,70)
           00 50 J=1,X
              DO 51 K=1, N
                  TD(J,X) = -1.3
              CONTINUE
51
              TD(J,J)=1.0
5.0
           CONTINUE
           00 52 3=1,8-1
              TD(ITOUR(J),ITOUR(J+1))=1.3
              TD (ITOUF (J+1), ITOUR (J)) =1.3
           CONTINUE
52
           TD(ITOUP(N),ITOUR(1))=1.0
           TD(ITOUF(!),ITOUP(!))=1.7
        RETURN
        END
        SUBPOUTINE SELCTY(I,C,G,NOD)
INTEGES I,C,M,K,COL
          INTECTE F, STN(5), STLA(5), ND(5), NCO
          FRAL G, TEMP, 5Y(5), SL_{5}(5), STOP, TAPPAY(7.1)
          COMMON /WALK/ M, ITOUR (71)
          COMMON /TEST/ TE(70,73)
          COMMON /INVT/ IT(140)
COMMON /DIST/ D(70,73)
          DO 85 J=1,8
          ITPRAY(J) = TD(ITOUR(2), J)
CONTINUE
65
          DO 57 K=1,5
              F = ?
              TEMP=0000.00
              DC 51 J=3, 11
                 IF(D(IT OUR(2), ITOUP(J)).CE.TEMP) COTOC!
                 IF (TATPAY(ITOUR(J)).CT.C.7) COTO61
                 TEMP=D(ITOUR(2), ITOUR(J))
                 E=3
61
             CONTINUE
              IF(F.NE. GYGOTOS4
             M=U-1
E=5
```

```
GOTO6 1
54
             TAFFAY(ITOUP(F))=1.0
             JY(K) = TEMP
             SLA(E) = D(ITOUP(1), ITOUR(2)) - D(ITOUP(F-1), ITOUR(1))
             STN(K) = ITCUP(F)
             STLA(K) = ITCUR(F+1)
             SLA(K)=SLA(K)+D(ITOUR(F),ITOUP(F-1))-SY(K)
             ND(E) = F - 1
             \mathbf{V} = \mathbf{V}
60
          CONTINUE
          STOP=0.7
          IF(M.EQ.G)STOP=9999.99
TEMP=SLA(1)
          COL=1
          DO 63 K=1,M
             IF(SLA(K).LF.TFMP)GOTO63
             TEMP=SLA(K)
             CCL=K
63
          CONTINUE
          C=SLA(COL)+STOP
          NOD=ND (COL)
          IT(2*I+1) = STY(CCL)
          IT(2*I+2)=STLA(COL)
       PETUPN
       END
        SUBROUTINE ADJTOR (I, P, NOD)
          INTEGER I,J,R,NOD
          COMMON /WALK/ N, ITOUP (71)
          COMMON /TDST/ TD(70,70)
          COMMON /INVT/ IT(140)
          COMMON /TMTF/ ITPTOP(140)
          OC 01 J=2,NCD
ITPTOR(MOD+2-J)=ITOUR(J)
          CONTINUE
31
          DO 82 J=2,110D
             ITOUF (J) = ITETOP (J)
          CONTINUE
          TD(IT(2*I-2),IT(2*I-1))=1.7
          TD(IT(2*I-1),IT(2*I-2))=1.7
          TD(IT(2*I-1),IT(2*I))=1.0
          TD(IT(2*I),IT(2*I-1))=1.0
          TD(IT(2*I),IT(1))=1.7
          R = 3
        PETURN
        END
```

```
SUBSOUTINE PEVICE (P)
          INTEGER P
          COMMON /WALK/ M, ITOUP (71)
          COMMON /TMIR/ ITPTOP (110)
          DO OT J=1,N+1
ITPTOF(N+2-J)=ITOUF(J)
9:
          CONTINUE
          DC 31 J=1, N+1
ITCUP(J)=ITPTCP(J)
91
          CONTINUE
          R = 1
        RETURN
        EMD
        FUNCTION PANDOM(A)
       REAL TEMP
       COMMON /PAND/ IX, IX, IZ
       IX=171*MOD(IX,177)-2*(IM/177)
        IY=172*MOD(IY, 176)-2*(IY/176)
       IC=170*MOD(IC,170)-2*(IC/170)
IF(IX.LT.0)IX=IX+30269
        IF(IY.LT.") IY=IY+30307
       IF(IZ.LT.?) IZ=IZ+30323
       TEMP=FLOAT(IX) /30259.0+FLOAT(IY) /30307.0+FLOAT(IC) /30022.0
       RANDOM=AMOD (TEMP, 1.0)
       RETURN
END
```

APPENDIX D

LISTING OF MATRIX UTILITIES PROGRAM

```
*** DISTANCE MATRIX UTILITIES ***
      IMPLICIT CHIRACTER*15 (A)
      INTECER IAMS
      COMMON /DIST/ D(199,199)
      COMMON /NAME/ ANAME(100)
      WRITE (*,1)
      WRITE (*,2)
      WRITE (*,3)
      WRITE(*,4)
WRITE(*,5)
12
      WRITE (*, 6)
      WRITE (*,7)
      WPITF (*,8)
      RRITE(*,9)
      WRITE (*, 10)
      READ(*,11) IANS
      IF((IANS.LT.1).OR.(IAMS.GT.6))GCTO12
      IF (IANS.EQ.1) CALL MATRIX()
      IF (IANS.EQ. 2) CALL EXTROT()
      IF(IANS.EQ.3)CALL CORECT()
      IF(IANS.ED.4)CALL MATPRT()
      IF(IANS.EQ.5)CALL DIST()
      IF (IANS.NE.6) COTO12
      STCP
1
      FCRMAT(1X,'This is a set of utilities to be used with')
2
      FORMAT(1X, the program ROUTE, RUN.
                                           They are to help ')
      FORMAT(IX, 'create and manipulate a distance matrix.')
      FORMAT(//lx,'l - create a distance matrix')
      FOFMAT(IX, '2 - extract a small matrix from a larger matrix')
      ECRMAT(IX,'3 - correct a previously stored matrix')
      FORMAT(IX,'4 - create a hardcopy of a distance matrix')
      FORMAT(1X,'5 - list names with their row number in matrix')
?
      FORMAT(IX,'6 - exit this program')
9
13
      FORMAT(//1X,'What is your selection? '\)
11
      FORMAT([1])
      END
      SUPPOUTINE MATRIX()
        IMPLICIT CHARACTER*15 (A)
        REAL DIST
        INTEGER N, I, J, L
        COMMON /DIST/ D(100,100)
        COMMON /NAME/ ANAME(100)
        WRITE(*,31)
        READ(*,32) U
        WRITE (*, 33)
```

```
DO 34 I=1, N
             WRITE(*,35) I
            READ(*,35) AMAME(I)
3 4
         CONTINUE
         WRITF(*,37) M,N
         WRITE(*,38)
         DC 39 I=1, N-1
            D(I,I) = 9999.99
            DO 40 J=I+1,N
                WPITE(*,41) I,J
                READ(*,42) DIST
                D(I,J)=DIST
DO 100 L=1,1000
100
                CONTINUE
                D(J,I)=DIST
                IF(DIST.GE.@)GCTO43
                J = !!
                I = N - 1
47
            CONTINUE
39
         CONTINUE
         D(N,N) = 9999.99
         CALL STORE (N)
       RETURN
31
         FORMAT(//lx,'How large is the problem? '\)
32
         FORMAT (BN, 13)
3.3
         FORMAT(//lx,'Input names for homeport and aids')
35
         FORMAT(1X, 'Mumber', 13, 'Name? '\)
3.5
         FCPMAT(A)
37
         FORMAT(1X, I3, 'BY ', I3, 'Matrix')
32
        FORMAT(' Input distances from point ( to point J')
FORMAT( Distance from pt ', I3, ' to pt ', I3, ' = '\)
41
42
         FORMAT (BN, F7.2)
      END
      SUBROUTINE EXTROT()
         IMPLICIT CHARACTER*15 (A)
         INTECER N.M.I.J. IFOW(100)
         PEAL SD(100,100)
         CHARACTER*15 ASYNM(100)
         COMMON /DIST/ D(100,100)
         CCMMON /NAME (103)
         WRITE (*, 13)
         WRITE(*,11)
         WPITE(*,12)
         WRITE(*,13)
        CALL OBTAIN(N)
```

```
#RITE(*,14)
         BF20(*,1=) M
         DC 23 I=1; 1
ARITE(*,21) I
             FORMAT(1X, 13, '. Row number from main matrix: '\)
21
             READ(*,15)IPOW(I)
DO 22 J=1,5
                SD(I,J) = D(IROW(I),J)
22
             CONTINUE
             ASMNM(I) = ANAMF(IROW(I))
2.3
         CONTINUE
         DC 23 J=1,N
DC 24 I=1,M
                D(I,J) = SD(I,IPOU(J))
24
             CONTINUE
             D(J,J) = 9900.03
             ANAME(J) = ASMNM(J)
23
         CONTINUE
         CALL STORE (M)
       PETURN
10
         FORMAT(//lx,'To create a smaller matrix from your main')
         FORMAT(1Y, 'listance matrix, enter the row numbers of')
11
         FORMAT(1%, 'main matrix ails which you desire in the ')
12
13
         FORMAT(IX, 'smaller ratrix')
14
         FORMAT(//lx,'; hat is the size of the smaller matrix? '\)
15
         FORMAT(DM, I3)
       EMD
       SUPPOUTINE CORDCT()
         IMPLICIT CHARACTER*15 (A)
INTECEP M,I,J,K,IAMS,IP,IC
         PENE DIST
         COMMON /DIST/ D(190,199)
COMMON /MAME/ ANAME(199)
         CALL OBTAIN (N)
         WRITE (*,14)
         WRITE (*,15)
16
         URITE (*,17)
         WPITE(*,19)
READ(*,10) INNS
         IF (IANS.EQ. 3) COTO 35
         IF (IAMS.FQ.2) COTO25
         IF (IANS.NF.1) GOTO16
         #PIRE(*,21)
         PEAD(*,12) IP
         WRITE (*,27)
```

```
READ(*,12) IC
         WRITE(*,23) IR,IC,D(IR,IC)
         NRITE (*,24)
         READ(*,13) DIST
         D(IR, IC) =DIST
         DO 100 K=1,1000
1.73
         CONTINUE
         D(IC, IR) =DIST
         GOTO16
25
         WRITE(*,26)
         WRITE(*,27)
         PEAD(*,12) IP
         WRITE(*,28)
         PEAD(*,12) IC
DC 30 I=IR,N-1
            DO 31 J=10, M
               WFITF(*,23) I,J,D(I,J)
               WRITE(*,24)
READ(*,13) DIST
                IF [DIST.GF.d.d) GOTO32
               I = N
                J=N
               GOTO31
32
               D(I,J) = DIST
                00 110 8=1,1000
110
               CONTINUE
               D(J,I)=DIST
31
            CONTINUE
            IC=I+2
            D(I,I)=0000.00
         ວດແຕ້ ໄດ້ບໍ່ຄົ
         COTOIS
        CHEL STORE(N)
35
      BETURN
12
        FORMAT (BN, 13)
13
        FORMAT(F7.2)
14
        FCRMAT(//1K, 'This utility is for correcting or corpleting')
15
        FORMAT(IX,'a proviously stored data file.')
1.7
        FC3MAT(LX,'1 - Single entries
                                                2 - Sequential entries!)
Īρ
        FORMAT(1X,'3 - Fxit corrections
                                                    Selection: '\)
19
        FORMAT(II)
        FOIMAT(1X, 'Pow number to correct '\)
        FORMAT(1%, 'Column number to correct 'N)
2.3
        FCPMAT(1X,'Dresunt listance from', 13,' to ',13,' = ',77.2)
24
        F MMAT(IX, 'Maw listance = 'N)
        ECOMAT(IV, 'To end entry of data, type a negative distance')
```

```
FORMAT(IN, 'Starting row number '\)
27
         EDEMAT(IX, 'Starting column number '\)
23
       END
       SUBROUTING MATERIA)
         IMPLICIT CHARACTER*15 (A)
         INTEGER I,J, N, IS, IE, JS, JE COMMON /DIST/ D(100, 100)
         COMMON /NAME/ ANAME(130)
         WPITE(*,3)
         WRITE(*,4)
         WRITE (*,5)
         NRITE (*,6)
         CALL CRIAIN(N)
         OPEN(3, FILE='MATPRT.DATA', STATUS='NEW', FORM='FORMATTED')
         IF=N
         IF (N.GT.50) IE=50
         JS=1
         JE=9
         DO P I=IS, IF
             IF(N.LT.10) JF=N
             DO 13 J=JS,JF
                WRITE(3,11) D(I,J)
10
             CONTINUE
             WRITE (3, 12)
         CONTINUE
3
         WRITE (3, 13)
         JS = JE + 1
         JE = JE + 9
         IF(JE.CT.N) JE=N
          IF(JS.LF.N) GCTO<sup>o</sup>
          IS = IE + 1
          IE=IE+53
          IF(IE.GT.N) IE=N
          IF(IS.LE.N) COTO?
          CLOSE(3)
          WPITE(*,14)
        RETURN
          FCRMAT(//lx, 'This utility propares a file containing the ')
          FORMAT(IX, 'distance matrix for the printir. You may have')
          FORMAT(1X,'to cut & past: some sections to det the matrix') FORMAT(1X,'in n by n form.')
 5
 6
          FORMAT(F9.2%)
 11
 12
          FORMAT('
 13
          FORMAT (1
```

```
FORMAT(' Use FORMAT command to optain print of MATERI. TATA')
14
      END
      SUBFOUTINE LIST()
         IMPLICIT CHAPACTER*15 (F)
        INTEGER W.I
        COMMON /DIST/ D(199,199)
        COMMON /NAME/ ANAME(103)
        WRITE (*,17)
        %RITE(*,11)
        WRITE (*, 12)
        WRITF(*,13)
        CALL OPTAIN(N)
        OPEN(3,FILE='LIST.DATA',STATUS='NFW',FORM='FORMATTED')
         DO 14 I=1,:
            WRITE(*,15) I,ANAME(I)
            WRITE (3,15) I, ANAME (I)
         CONTINUE
14
         CLOSE(3)
      RETURN
        FORMAT(//lx,'Inis routine will list aid row number and')
FORMAT(lx,'aid name. The list will scroll by the ')
10
11
12
         FORMAT(IX, 'screen and be saved to disk under file
         FORMAT(IX, 'LIST. DATA use FORMAT command for hardcopy')
13
15
            ECRMAT(1X, 13, 5X, A)
      END
       SUPPOUTINE OPTAIN(N)
         IMPLICIT CHAPACTER*15 (A)
         INTEGER N.I.J
CHAPACTER*15 ENAME
         COMMON /DIST/ D(100,100)
         COMMON / MAME/ ANAME (100)
         WPITT(*,13)
         PEAD(*,11) FMAME
         OPEN(4, FILE = FYAME, FORM= 'FORMATTED')
         READ(4,12,\Gamma\Gamma\GammaP=14) N
         READ(4,13,EFF=14)((O(I,J),J=1,U),I=1,Y)
         READ(4,11,EPP=14)(AMAMF(I),I=1,N)
         CL^3F(4)
       RETURY
14
         MRITE(*,15)
         CLOSE(4)
       STOP
         FORMAT(//IX, 'Under what file is the distance matrix? 'N)
1 3
11
         FORMAT(A)
```

```
FORMAT(IB)
12
         F `FM T(F7.2)
1.3
         FORMAT(1X, 'Error luring load')
15
       END
       SUPPOSITIVE STORF (M)
         IMPLICIT CHARACTER*15 (A)
          INTEGER ",I,J
         CHARACTER*15 FNAME
COMMON /DIST/ D(123,100)
          COMMON /NAME/ ANAME(130)
         WRITE(*,10)
READ(*,11) ENAME
         WFITE(*,12) ENAME
         OPFN(3, FILE=FNAME, STATUS='MEW', FORM='FORMATTID')
         WRITE(3,13,FPP=16) N
         \text{WFIFE}(3,14,\text{ERP}=16)((D(I,J),J=1,N),I=1,N)
         WRITE(3,11, EFF=16) (ANAME(I), I=1, M)
         CLOSE(3)
         WFITE (*,15)
          COTOLS
1.5
         WRITE(*,17)
18
          CLOSE(3)
       PETURN
10
         FORMAT(//lx,'File name for matrix to be saved? '\)
11
         FORMAT(' The file will be saved under ',\Delta) FORMAT(I3)
1.4
          FORMAT(E7.2)
         TORMAT(IM,'File saved without error')
FORMAT(IM,'Error while saving file')
15
```

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